Time variation of G and α within models with extra dimensions

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Abstract. We derive the formulae for the time variation of the gravitational "constant" G and of the fine structure "constant" α in various models with extra dimensions and analyze their consistency with the available observational data for distant supernovae. We find that the reported variation of α translates into a small variation of G that makes distant supernovae to appear *brighter*, in contradiction with recent observations of high z supernovae. The significance of these results within the framework of some cosmological scenarios is also discussed. We find, however, that the magnitude of the effect is not large enough to safely discard the models with extra dimensions studied here.

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1. Introduction

One of the most challenging issues of modern physics is the quantization of the gravitational interaction. Within this theoretical framework it is worth noticing that although the controversial issue of the variation of the fundamental couplings dates back to the large-number hypothesis of Dirac [1], it has recently become a subject of intensive experimental and theoretical studies — see [2] for an excellent review. Modern theories, like the string/M-theory or brane models, do not necessarily require a variation of the fundamental constants but they provide a natural and self-consistent framework for such variations. Most of such theories assume the existence of additional dimensions of the space-time and, therefore, contain a built-in mechanism allowing for the time variation of the observed couplings in four dimensions. This feature is rather easy to understand. Within the multidimensional approach the interactions are described by a fundamental

theory formulated in $4 + d$ dimensions, and the conventional four-dimensional theory appears as a result of the dimensional reduction. Couplings in four dimensions are determined by a set of a few constants of the multidimensional theory and the size R of the space of extra dimensions. The multidimensional constants are assumed to be genuinely fundamental and, consequently, do not vary with time. On the other hand, in the cosmological or astrophysical context it is natural to assume that R varies with time, very much in the same way as the scale factor of the three-dimensional space does. This leads to the time variation of parameters of the effective four-dimensional theory, like the gravitational constant G and fine structure constant α [3]–[7]. Moreover, since their time dependences are given by the same scale factor R these variations may turn out to be correlated, and it is natural to wonder what kind of relation is predicted in multidimensional scenarios. The effect of these correlations for various couplings in the framework of theories of unification has been discussed in [8]. It has also been suggested that these correlations may unveil the mechanism of the time variation of the couplings and, in this way, give insight into the underlying fundamental theory. It is important to emphasize that a variation of the fundamental couplings may lead to deeper consequences or effects in the theory of particle interactions. In particular, it has been shown that a variation of fundamental couplings can be intrinsically related to the Lorentz and CPT violation [9].

There have been several attempts to measure the rate of variation of the fundamental constants but many of them have yielded just upper bounds on the absolute value of the rate of change. However, very recently, the rate of variation of the fine structure constant has been measured using high resolution spectroscopy of QSO absorption systems. To be specific a non-zero detection of the variation of the fine structure constant, $\Delta\alpha/\alpha \sim -10^{-5}$ at $z \sim 1.5$, has been reported [10]. Although this result is still the subject of strong debate it is worth studying its theoretical and observational consequences. In addition, the observations of high redshift ($z > 0.1$) Type Ia supernovae [11, 12] and the analysis of the CMB strongly suggest a flat, $\Omega_R = 0$, Universe [13], with a mass density $\Omega_M \simeq 0.3$ and a non-vanishing cosmological constant $\Omega_\Lambda \simeq 0.7$. This result, in turn, motivated a considerable amount of papers looking for viable alternatives. Amongst these alternatives the variability of the gravitational constant was one of the proposed alternatives [14] to reconcile the observational data of distant supernovae with an open $\Omega_\Lambda = 0$ Universe.

As it has been argued in [15], models in which the time variation of couplings — and in particular of the fine structure constant — is generated by the dynamics of a cosmological scalar field face serious difficulties. Namely, using rather general arguments, these authors show that in order to explain the observations of [10] such models require an extremely precise fine-tuning that cannot be explained by any known mechanism. The problem is essentially the huge back-reaction produced by varying couplings on the vacuum energy. This difficulty could be intimately related to the long-standing cosmological constant problem. Hence, its satisfactory solution could also provide a mechanism to suppress the enormous variation of the vacuum energy due to

the time variation of α, but this is beyond the scope of this paper. Having adopted this point of view, we do not discuss this issue furthermore in the present work. Instead, the aim of the our work is to derive the relations between $\dot{G}/G$ and $\dot{\alpha}/\alpha$ in various models in $(4+d)$ space-time dimensions and to confront them with the observational data, namely to compare the detected variation of the fine structure constant [10] with the existing bounds on the variation of G at cosmological distances. The models we are going to consider include the classical Kaluza-Klein models — see [16, 17] for reviews on the subject — models with multidimensional gravitational and Yang-Mills fields [18, 19], and the Randall-Sundrum model with two branes and gauge and fermionic fields propagating in the bulk [20]–[22]. The paper is organized as follows. In Section 2 formulae for the time variation of G and α in various models are derived. In Section 3 we use our theoretical results and the experimental value for $\dot{\alpha}/\alpha$ [10] to obtain an estimate of the rate $\dot{G}/G$, which is then confronted with the data obtained from distant Type Ia Supernovae [11, 12]. Finally, in Section 4 conclusions and some discussion of the results are presented.

2. Time variation of couplings in multidimensional theories

In this section we consider theories with extra spatial dimensions. For the sake of simplicity we assume that the geometry of the space of additional dimensions is described by just one scale factor R. In most of the cases the generalization to geometries with a few scales is straightforward and does not bring any qualitatively new features to the effect we are going to study. Moreover, the theories studied here are assumed to be part of a cosmological scenario with varying R. The specific form of the function $R(t)$ depends on the details of the scenario — see, for instance, [23]. Here we simply assume that the evolution of the multidimensional Universe at the cosmological scale is described by some scenario which predicts certain background metric with varying scale $R(t)$. Another possible class of scenarios would be the ones postulating a first order phase transition in the Universe. However, we will not consider this last possibility here and we limit ourselves to multidimensional theories describing the time variation of G and α which fulfill the following conditions: (1) the four-dimensional effective theory — that is, the dimensionally reduced theory — includes both Einstein gravity and Maxwell electrodynamics; and (2) the time variation of the scale factor $R(t)$ of the space of extra dimensions leads to the time variation of both the gravitational constant G and the fine structure constant α. To calculate $\dot{G}/G$ and $\dot{\alpha}/\alpha$ we focus only on the gravitational and electromagnetic sectors of the reduced theory. These time variations as well as that of $R(t)$ are supposed to be slow enough in comparison with the phenomena described by these sectors — namely, the electromagnetic processes at the microscopic level. Our analysis will be rather general and will not rely on any particular function $R(t)$.

2.1. Kaluza-Klein theories

In two pioneering papers [24] T. Kaluza and O. Klein formulated the essential elements of the multidimensional approach to the description of fundamental interactions which was later called the Kaluza-Klein approach. They considered the pure Einstein gravity in five-dimensional space time $M^4 \times S^1$ described by the multidimensional metric tensor $\hat{g}_{MN}$ and showed that the sector of zero modes of the dimensionally reduced theory includes the four-dimensional gravity and Maxwell theory. Here M^4 is the four-dimensional Minkowski space-time, and S^1 is the circle. Later this construction was generalized [25] to more general compact spaces of extra dimensions. In these cases the reduced theory contains Einstein gravity and Yang-Mills fields with the gauge group determined by the isometry group of the space of extra dimensions. In the standard setting the multidimensional Kaluza-Klein theory is the pure Einstein theory on $M_4 \times K_d$ with the action given by

$$S = \int d^{4+d}\hat{x}\sqrt{-\hat{g}}\frac{1}{16\pi G_{(4+d)}}\mathcal{R}^{(4+d)}. \tag{1}$$

Here M_4 is a (curved) four-dimensional space-time, K_d is a compact manifold of extra dimensions, $\hat{g} = \det \hat{g}_{MN}$, $(M, N = 0, 1, 2, \ldots, 3 + d)$, $\mathcal{R}^{(4+d)}$ is the scalar curvature in $M_4 \times K_d$, and $G_{(4+d)}$ is the multidimensional gravitational constant, which is assumed to be truly constant and does not depend on time. According to the procedure of dimensional reduction, to obtain the four-dimensional effective theory, firstly the $\mu\nu$-components $(\mu, \nu = 0, 1, 2, 3)$ of the metric tensor are identified as the four-dimensional metric tensor. Certain combinations of the rest of the components, $\hat{g}_{\mu m}$, $\hat{g}_{n\nu}$ and $\hat{g}_{mn}$ with $m, n = 4, \ldots, 3 + d$ are identified as gauge field multiplets A_μ and scalar fields ϕ_{mn}. Secondly, the mode expansion of all these fields is performed — see, for example, [17]. The coefficients of the expansion depend only on x^μ and are interpreted as four-dimensional fields. In general there is an infinite number of them but here we are interested in the sector containing only zero modes of the mode expansion. Its action is given by

$$S_0 = \int d^4x \left[\frac{1}{16\pi G(t)}\mathcal{R}^{(4)} + \sum_i \frac{1}{4g_i(t)^2} Tr F^{(i)}_{\mu\nu} F^{(i)\mu\nu}\right], \tag{2}$$

where $G(t) \equiv G_{(4)}(t)$ is the four-dimensional gravitational constant. The parameters $g_i(t)$ are the gauge couplings, and the index i labels the simple subgroups of the gauge group. The dimensional reduction of the initial Kaluza-Klein action S, given by Eq. (1), yields in addition to S_0, given by Eq. (2), terms including non-zero modes of the gravitational, gauge and scalar fields as well as terms proportional to $(\dot{R}/R)^2$. The scalar fields usually give highly non-linear interaction terms and are coupled non-minimally to the gravitational and gauge fields. For the sake of simplicity, the scalar fields are supposed to be frozen out and their contribution is neglected. Identifying the gravitational and gauge couplings from the action S_0 for the zero modes we obtain the following expressions for $G(t)$ and $g_i(t)^2$ in terms of $G_{(4+d)}$ and the radius $R(t)$ of the

space of extra dimensions:

$$G(t) = \frac{G_{(4+d)}}{V_d(t)}, \tag{3}$$

$$g_i^2(t) = \tilde{\kappa}_i \frac{G_{(4+d)}}{R(t)^2 V_d(t)}, \tag{4}$$

where $V_d(t) \sim R(t)^d$ is the volume of the space of extra dimensions and $\tilde{\kappa}_i$ are coefficients which depend on the isometry group of K_d. Equations (3) and (4) should be regarded as leading order approximations. The terms omitted in the reduced action will give sub-leading corrections, in particular through loop effects. As it has been said above, we assume that the dimensionally reduced theory includes the electrodynamics. Then the fine structure constant $\alpha(t)$ is given by a linear combination of $g_i^2(t)$, the specific relation depending on the model, in particular on the gauge group and the scheme of the spontaneous symmetry breaking. From Eq. (4) it follows that

$$\alpha(t) = \kappa_1 \frac{G_{(4+d)}}{R(t)^2 V_d(t)}, \tag{5}$$

where κ_1 is some constant. Therefore, the time variation of G and α is determined by the function $R(t)$. Since $\dot{V}_d/V_d = d(\dot{R}/R)$, we get

$$\frac{\dot{G}}{G} = -\, d\frac{\dot{R}}{R}, \tag{6}$$

$$\frac{\dot{\alpha}}{\alpha} = -\,(d+2)\frac{\dot{R}}{R}. \tag{7}$$

As a consequence the time variation of the fine structure and gravitational constants are related by

$$\frac{\dot{\alpha}}{\alpha} = \frac{d+2}{d}\frac{\dot{G}}{G}. \tag{8}$$

2.2. Einstein-Yang-Mills theories

Consider a theory in the $(4+d)$-dimensional space-time $M_4 \times K_d$ that includes gravity and the Yang-Mills field with the action

$$S = \int d^{4+d}\hat{x}\sqrt{-\hat{g}}\left[\frac{1}{16\pi G_{(4+d)}}\mathcal{R}^{(4+d)} + \frac{1}{4g^2_{(4+d)}} Tr\hat{F}_{MN}\hat{F}^{MN}\right], \tag{9}$$

where, as above, $G_{(4+d)}$ is the multidimensional gravitational constant, and $g_{(4+d)}$ is the multidimensional gauge coupling. Both are supposed to be constant in time. The dimensionally reduced theory includes the Einstein gravity and the four-dimensional gauge fields with an action similar to that of Eq. (2), and, in addition, scalar fields with a quartic potential. The explicit form of the dimensionally reduced theory depends on the topology and geometry of the space of extra dimensions and the multidimensional gauge group. The case of K_d being a homogeneous space was studied in detail in

the literature [18] — see also [19] for reviews on the subject. The four dimensional gravitational constant is given by

$$G(t) = \frac{G_{(4+d)}}{V_d(t)}, \tag{10}$$

We assume that the gauge part of the initial multidimensional model is such that its dimensional reduction gives the bosonic sector of the electroweak Glashow-Salam-Weimberg model in four dimensions. Examples of this kind can be found in [19]. Then the fine structure constant in the reduced theory is related to the multidimensional gauge coupling $g_{(4+d)}$ by

$$\alpha(t) = \kappa_2 \frac{g^2_{(4+d)}}{V_d(t)}. \tag{11}$$

where κ_2 is some constant factor. From these expressions the following relation between the time variations of G and α can be easily obtained:

$$\frac{\dot{\alpha}}{\alpha} = \frac{\dot{G}}{G}. \tag{12}$$

We would like to mention here that the same realtion appears in a ten-dimensional model obtained as a low energy limit of a string theory [6].

2.3. Randall-Sundrum model

A different multidimensional setting motivated by string/M-theories was proposed and studied in [20]. The model is formulated in the five-dimensional space-time $M^4 \times K_1$ with the fifth dimension compactified to the orbifold $K_1 = S^1/Z_2$ of radius R — see [26] and [27] for reviews on the subject. In the initial version of the Randall-Sundrum model with two branes located at the fixed points of the orbifold, known also as the RS1 model, only gravity propagates in the five-dimensional bulk. The background metric solution is given by

$$ds^2 = e^{-2kR(|\phi|-\pi)}\eta_{\mu\nu}dx^\mu dx^\nu + R^2 d\phi^2 \tag{13}$$

[20, 28, 29], where ϕ is the coordinate of the orbifold ($0 \leq \phi \leq \pi$), $\eta_{\mu\nu}$ is the four-dimensional Minkowski metric tensor, and $k > 0$ is a parameter of the dimension of mass, its value being fixed by the brane tensions. Our three-dimensional space is identified with the brane with negative tension at $\phi = \pi$. The fields of the Standard Model are assumed to be localized at this brane. Due to the warp factor in the background metric, Eq. (13), the RS1 model provides an elegant geometrical solution to the hierarchy problem [20]. In addition it opens new phenomenological possibilities like the observation of higher-dimensional gravity effects in the current or future collider experiments [30].

The reduction formula expressing the four-dimensional Planck mass in terms of the fundamental (five-dimensional) mass scale $M = (16\pi\hat{G}_{(5)})^{-1/3} \sim k$ is derived [29] using a metric given by Eq. (13), which corresponds to the Galilean coordinates on the physical brane at $\phi = \pi$. One gets

$$M^2_{Pl} = \frac{M^3}{k}\left[e^{2\pi kR} - 1\right] \tag{14}$$

(see also [27]). To generate the correct hierarchy between the Planck scale and the TeV-scale the product kR must be $kR \approx 11 \div 12$. As before, let us now consider this model as a part of a more general cosmological scenario with a slow varying scale factor $R(t)$. The background solution (13) must be modified accordingly, in particular its four-dimensional part takes the form of the Robertson-Walker metric with the scale factor $a(t)$ multiplied by the conformal warp factor including $R(t)$. Models of this type have been extensively studied in the literature — see, for example, [31]. It is natural to expect that the reduction formula in the first approximation remains the same as in Eq. (14) but with $R = R(t)$. From this equation it readily follows that

$$G(t) = \frac{k}{16\pi M^3} \frac{1}{e^{2k\pi R(t)} - 1} \approx \frac{k}{16\pi M^3} e^{-2k\pi R(t)}. \tag{15}$$

The time variation of the Newton constant is then given by

$$\frac{\dot{G}}{G} = -2\pi k R(t) \frac{\dot{R}}{R} \frac{1}{1 - e^{-2k\pi R(t)}} \approx -2\pi k R(t) \frac{\dot{R}}{R} \tag{16}$$

However, since the fields of the Standard Model are localized on the brane and do not depend on R the RS1 model does not contain a simple mechanism for the variation of the fine structure constant. To describe this effect one has to consider bulk gauge and, perhaps, fermionic fields. Such models have been studied in a number of papers, see for example [21, 22]. We assume that, similar to the case of the Einstein-Yang-Mills theories studied in §2.2, the bulk gauge fields yield the gauge (and perhaps the scalar) sector of the Standard Model on the brane. The electromagnetic $U(1)$-field appears in a usual way as a part of this sector after the spontaneous symmetry breaking. Similar to Eq. (11), the fine structure constant turns out to be related to the multidimensional gauge coupling $g_{(5)}$ by

$$\alpha(t) = \kappa_3 \frac{g_{(5)}^2}{R(t)}, \tag{17}$$

where κ_3 is some constant. From Eqs. (16) and (17) we obtain the following relation:

$$\frac{\dot{\alpha}}{\alpha} = \frac{1}{2\pi k R(t)} \frac{\dot{G}}{G} (1 - e^{-2k\pi R(t)}) \approx \frac{1}{2\pi k R(t)} \frac{\dot{G}}{G}. \tag{18}$$

We would like to mention at this point that as it was observed in [7] the effect of the time variation of couplings in brane-world scenarios is closely related to the resolution of the hierarchy problem.

2.4. General remarks

Let us make a few remarks. Expressions (5), (11), and (17) are classical, or tree-level relations. They define the fine structure constant $\alpha(M_R; t)$ at the scale $M_R = R^{-1}(t)$. We have found that $\dot{\alpha}(M_R; t)/\alpha(M_R; t) = \rho(\dot{R}/R)$, where ρ is a constant. In order to relate $\alpha(M_R; t)$ to the correspondig value $\alpha(\mu; t)$ at some low energy scale μ, for example at the electroweak scale $\mu = M_Z$, one should take quantum corrections into

account by using the renormalization group formulas for running couplings. By standard considerations one obtains a relation of the form

$$\frac{1}{\alpha(\mu;t)} = \frac{1}{\alpha(M_R;t)} + A\ln(\mu/M_R),$$

where A is a constant of order one [32]. Though, in principle, the second term also contributes to the time variation of $\alpha(\mu;t)$, in fact its variation is dominated by the first term, $\alpha^{-1}(M_R;t)$. The reason is quite simple. Let us take the point of view that the scale μ does not vary with time. In other words, the variation of the dimensionless quantity $\mu/M_R(t)$ is determined by $R(t)$. Then by a straightforward time derivation of the previous expression one obtains:

$$\frac{\dot\alpha(\mu,t)}{\alpha(\mu;t)} = \rho\frac{\dot R}{R} - \alpha(\mu;t)A\frac{\dot R}{R}\big[1+\ln\mu R(t)\big]. \tag{19}$$

The first term in this expression is the time variation calculated in §2.1, 2.2 and 2.3 for three specific classes of models; the second term is of order $\mathcal{O}(\alpha)$. Hence, it is sub-dominant and will be neglected from now on. If the scale μ is defined in a different way and appears to be time dependent, the form of the second term in Eq. (19) may change, but it will remain to be sub-dominant. A similar analysis was presented in [4] and [8].

In order to summarize, Eqs. (8), (12), and (18) can be written in the following general form:

$$\frac{\dot\alpha}{\alpha} = \beta(R)\frac{\dot G}{G}, \tag{20}$$

where

$$\beta(R) = \begin{cases} \frac{d+2}{d} & \text{for the Kaluza-Klein theories,} \\ 1 & \text{for the Einstein-Yang-Mills theories,} \\ \frac{1}{2\pi k R(t)} & \text{for the Randall-Sundrum-type model.} \end{cases} \tag{21}$$

Note that $\dot\alpha/\alpha \propto \dot G/G$, that the constant of proportionality is positive, and that $\beta\sim 1$ in the case of the Kaluza-Klein and Einstein-Yang-Mills theories, and $\beta\sim 10^{-2}$ in the case of the Randall-Sundrum model. We would like to emphasize that the result given in Eqs. (20) and (21) is robust, since does not depend on details of the models and the specific form of $R(t)$.

If the dimensional reduction of the multidimensional models gives a four-dimensional model of unification, then an analysis similar to that carried out previously in subsections 2.1–2.3 gives the time variation of a single coupling constant $\alpha_{\rm GUT}$. In this case, Eq. (19) for the coupling running is different and relates the three couplings of the Standard Model with $\alpha_{\rm GUT}$. The electromagnetic coupling at the low energy scale μ is calculated in a standard way. Making an analysis similar to that of [8] it can be shown that the time variation in the leading approximation is related to $\dot\alpha_{\rm GUT}/\alpha_{\rm GUT}$ by:

$$\frac{\dot\alpha(\mu;t)}{\alpha(\mu;t)} \propto \frac{\dot\alpha_{\rm GUT}}{\alpha_{\rm GUT}}$$

where the proportionality constant is of order unity.

Finally, let us also mention that there is a class of models with branes and large extra dimensions, the Arkani-Hamed-Dimopoulos-Dvali (ADD) models, which also provide a solution to the hierarchy problem and predict new observable effects with massive gravitons [33]. However, similar to the case of the RS1-model, the ADD-model in its standard version with only the gravitational field propagating in the bulk does not describe the time variation of the fine structure constant.

3. Observational constraints on $\dot{\alpha}/\alpha$ and $\dot{G}/G$

The detailed analysis of three distant ($z \sim 3.5$) quasar absorption line data sets has provided for the first time direct evidence that the fine structure constant α was *smaller* in the past [10]. In particular, the detection of a variation in α:

$$\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha(z) - \alpha_0}{\alpha_0} = (-0.66 \pm 0.11) \times 10^{-5}, \tag{22}$$

where $\alpha_0 = \alpha(z_0)$ is the present day value has been reported. It is of importance for our analysis to realize that $\Delta\alpha = \alpha(z) - \alpha_0 < 0$. We also emphasize that, since the error bars are considerably smaller than the reported value, this is not just an upper bound but a direct measurement of the rate of change of the fine structure constant. Whether this detection is genuine or is affected by systematic errors is still a matter of debate [2, 34]. Using Eqs. (20) and (21) it turns out that, provided that the observational determination of $\Delta\alpha/\alpha$ is correct, the variation of $\dot{G}/G$ in the models studied in the previous section is *positive* and we face a *smaller* value for G in the past. Given a typical age of the Universe $\tau_U \sim 14$ Gyr and assuming a constant rate of change it is straightforward to derive from Eq. (22) the following estimate

$$\dot{G}/G \sim +10^{-15} \text{ yr}^{-1} \tag{23}$$

for the Kaluza-Klein and the Einstein-Yang-Mills theories, whereas it is a factor of 10^2 larger for the Randall-Sundrum scenario.

Let us see if this estimate matches the currently available experimental bounds on the variation of G. Several constraints on the *local* rate of change of G using the observation of lunar occultations and eclipses, planetary and lunar radar-ranging measurements, the evolution of the Sun, gravitational lensing, Viking landers or data from the binary pulsar PSR 1913+16 have been obtained up to now. First, it is important to realize that all these measurements give just upper bounds on the rate of variation of G. Among these measurements, the last one provided for many years the most reliable upper bound [35]

$$-(1.10 \pm 1.07) \times 10^{-11}\text{yr}^{-1} < \dot{G}/G < 0.$$

However, the best upper bound has been obtained using helioseismological data [36]:

$$-1.6 \times 10^{-12}\text{yr}^{-1} < \dot{G}/G < 0.$$

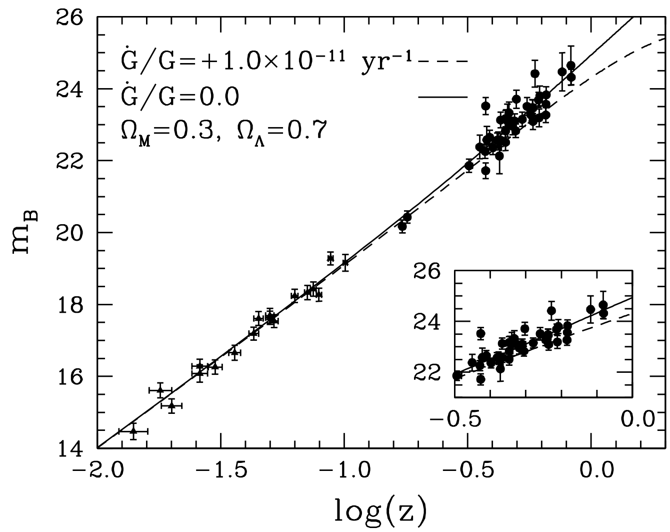

Figure 1. The Hubble diagram of distant supernovae, assuming the preferred cosmological scenario of the SCP, for $\dot{G}/G = 0$ (solid line), $\dot{G}/G = +10^{-11}$ yr^{-1} (dashed line). For $\dot{G}/G = +10^{-15}$ yr^{-1} the results are indistinguishable from those of the case with constant G. The observational data are taken from [11]. The inset shows an enlarged view of the region around $z \sim 0.5$. See the text for further details.

Note that all these upper bounds are local. At *cosmological distances* the best upper bound for the rate of variation of G comes from the Hubble diagram of distant type Ia supernovae. By taking into account that distant Type Ia supernovae appear to be *dimmer* than local supernovae [11, 12], the following upper bound on the variation of G was obtained [37]:

$$-10^{-11}\ \mathrm{yr}^{-1} \lesssim \dot{G}/G < 0 \quad \text{at} \quad z \simeq 0.5.$$

We would like to stress that all these upper bounds — regardless if they are local or obtained at moderately high redshifts — are negative and, consequently, positive values of $\dot{G}/G$ seem to be not allowed by the present astrophysical data. Consequently, these upper bounds are then *at odds* with the estimate given by Eq. (23) which was obtained from the recent determination of $\dot{\alpha}/\alpha$, Eq. (22), within the multidimensional models studied in Sect. 2. In the following we will elaborate on this. It is as well worth mentioning at this point that the Oklo natural nuclear reactor [38, 39] severely constrains a variation of α but, again, at low z.

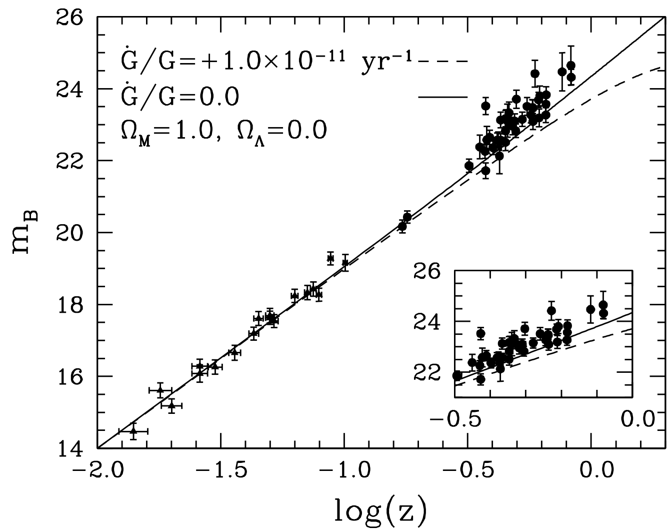

Figure 2. Same as in Fig. 1, but assuming a flat, matter-dominated universe, $(\Omega_M, \Omega_\Lambda) = (1.0, 0.0)$.

Fig. 1 shows the Hubble diagram of distant Type Ia supernovae for three values of $\dot{G}/G$ and for the preferred scenario of the Supernova Cosmology Project, namely $(\Omega_R, \Omega_M, \Omega_\Lambda) = (0.0, 0.3, 0.7)$. For the sake of simplicity we have assumed that $\dot{G}/G$ remains constant within this z interval. As it can be seen, the effect of a *positive* value of $\dot{G}/G$ is to make distant supernovae to appear *brighter*. This behavior is just the opposite to what it is observationally found. However, as it can be seen in Fig. 1, the effect of a varying G at a rate given by Eq. (23), which is the value derived from the observed variation of α, is practically indistinguishable from that of $\dot{G}/G = 0$. For the purpose of illustration we also show the curve with $\dot{G}/G = +10^{-11}$ yr^{-1}. Note also that the case $\dot{G}/G \simeq +10^{-13}$ yr^{-1} predicted in the Randall-Sundrum-type model — see Eq. (21) — is bracketed by these two curves. There are other scenarios which have been generally used to explain an accelerating universe without the need of invoking a non-vanishing cosmological constant by attributing this behavior to the presence of extra dimensions. To this regard in Fig. 2 we show the Hubble diagram for a flat matter dominated universe $(\Omega_R, \Omega_M, \Omega_\Lambda) = (0.0, 1.0, 0.0)$. As it can be seen in this figure there is no possible way to reconcile the multidimensional models considered in Sect. 2 with the observations no matter which the value of $\dot{G}/G \geq 0$ is.

The conclusions above can be put into a quantitative form. Using the 2σ confidence contours for $z = 0.5$ obtained from the fit to the Hubble diagram of Type Ia supernovae given in [37] we have calculated bounds to $\dot{G}/G$. For the currently favored cosmological scenario $(\Omega_R, \Omega_M, \Omega_\Lambda) = (0.0, 0.3, 0.7)$ and for the flat matter dominated case $(\Omega_R, \Omega_M, \Omega_\Lambda) = (0.0, 1.0, 0.0)$ we have obtained the following estimates respectively:

$$\begin{array}{ll} \Omega_R = 0.0,\ \Omega_M = 0.3,\ \Omega_\Lambda = 0.7 & -1.4 \cdot 10^{-11}\ \mathrm{yr}^{-1} < \dot{G}/G < +2.6 \cdot 10^{-11}\ \mathrm{yr}^{-1}, \\ \Omega_R = 0.0,\ \Omega_M = 1.0,\ \Omega_\Lambda = 0.0 & -2.9 \cdot 10^{-11}\ \mathrm{yr}^{-1} < \dot{G}/G < -0.3 \cdot 10^{-11}\ \mathrm{yr}^{-1}. \end{array}$$

Here, as before, we used the typical value $\tau_U = H_0^{-1} = 14$ Gyr and assumed a constant rate of change of G. As it has been already discussed above, the scenario $(\Omega_R, \Omega_M, \Omega_\Lambda) = (0.0, 1.0, 0.0)$ does not seem to allow a positive $\dot{G}/G$. The cosmological scenario with $(\Omega_R, \Omega_M, \Omega_\Lambda) = (0.0, 0.3, 0.7)$, which is the preferred scenario of the SCP, is among the allowed ones. It can be shown that for a flat Universe positive values of $\dot{G}/G$ are allowed only if $\Omega_\Lambda \gtrsim 0.15$. Another interesting case is, for instance, $(\Omega_R, \Omega_M, \Omega_\Lambda) = (0.5, 0.5, 0.0)$, for which we obtain

$$-2.3 \cdot 10^{-11}\ \mathrm{yr}^{-1} < \dot{G}/G < +0.3 \cdot 10^{-11}\ \mathrm{yr}^{-1}.$$

at the 2σ confidence level.

4. Conclusions and caveats

We have derived the formulae for the time variation of the gravitational "constant" G and of the fine structure "constant" α for three classes of models with extra dimensions. We have found that such variations are related and we have derived the explicit relation — Eqs. (20) and (21) — which does not rely on the specific form of the time dependence of the scale factor of extra dimensions $R(t)$. For the classes of models considered in §2 the relative sign between $\dot{\alpha}/\alpha$ and $\dot{G}/G$ turns out to be *model independent*. Then, using these expressions and the reported variation of α based on the available data obtained from distant QSOs [10], we have derived the estimate $\dot{G}/G \sim +10^{-15}\ \mathrm{yr}^{-1}$. This value of the time variation of the gravitational constant makes distant supernovae to appear *brighter*, in contrast with observations. However, the effect is too small to safely discard the classes of models with extra dimensions considered here. In fact, a positive rate of variation $\dot{G}/G \sim +1 \cdot 10^{-11\pm1}\ \mathrm{yr}^{-1}$ has been predicted within a $N = 1$ ten-dimensional supergravity and with non-dynamical dilaton [5]. However, and as it has been shown in [6], when the dilaton dynamics are taken into account the rate of change of α is too small to be observed. Let us mention that in this case the relation between $\dot{\alpha}/\alpha$ and $\dot{G}/G$ appears to be given by Eq. (12).

We have also computed the Hubble diagrams of distant Type Ia supernovae in the framework of these models for several typical cosmological scenarios and analyzed their consistency with the available observational data. We have found that, if a flat Universe is assumed, models with extra dimensions and *positive* values of $\dot{G}/G$, in accordance with Eq. (22), can reproduce the observational data only if $\Omega_\Lambda \gtrsim 0.15$. One should

however keep in mind that provided that the value of $\dot{\alpha}/\alpha$ derived from QSOs turns out to be a genuine detection and given that the corresponding estimate of $\dot{G}/G$ is very small, at the redshifts of interest ($z \sim 1.5$) the deviation with respect to the Hubble diagram for $\dot{G}/G = 0$ is also very small so the observations still leave much room for the multidimensional models studied here provided that $\Omega_\Lambda \gtrsim 0.15$. Conversely, for the class of models studied here to be able to reproduce the observational data with $\Omega_\Lambda \lesssim 0.15$ *negative* values of $\dot{G}/G$ are needed (see Fig. 2). However, the absolute value of $\dot{G}/G$ needed to fit the observations should be much larger than that obtained here from distant QSOs.

A natural question which arises is whether the robustness of the observational results on $\dot{\alpha}/\alpha$ and $\dot{G}/G$ allows to safely discard the multidimensional approach for scenarios involving a flat Universe with $\Omega_\Lambda \lesssim 0.15$, attending *exclusively* to the relative signs of $\dot{\alpha}/\alpha$ and $\dot{G}/G$. There are a few possibilities. One of them is that the observational data is not precise enough at the moment and does not allow to draw any definite conclusions. This is perhaps the simplest and most obvious explanation. The situation may improve when, for example, better or additional observational determinations of $\dot{\alpha}/\alpha$ are obtained or when experimental data from distant supernovae become available in the future (with missions like SNAP) to give more accurate bounds on $\dot{G}/G$. Although this is indeed the most straightforward explanation, let us consider other possibilities. Firstly, it may turn out that our consideration of the time variation of the fundamental couplings is too rough and some effects which modify Eq. (20) are missing. And secondly, it could be that the multidimensional models considered here may also be too simple and phenomenologically unsatisfactory and should be substituted by more elaborated ones. In any case, questioning the applicability of the multidimensional approach for the description of the fundamental interactions should be preceded by additional more detailed theoretical studies and should be confronted with more accurate observational data.

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